Trig Equations

The preliminary goal in solving a trig equation is to isolate the trig function first.

Example: Solve $1-2\cos x=0$.

Isolate the cos x term like you would isolate any variable term in an algebraic equation.

$$1-2\cos x=0$$
$$-2\cos x=-1$$
$$\cos x=\frac{1}{2}$$

We know that if our angle is from 0 to 2π, we have

$$x=\frac{\pi}{3} \text{ or } x=\frac{5\pi}{3}$$

But there are many angles that are coterminal with these 2 angles. Every time we add a multiple of 2π we get a coterminal angle. Thus, the solution should be

$$x=\frac{\pi}{3}+2n\pi \text{ or } x=\frac{5\pi}{3}+2n\pi$$

Example: Solve $\sin x + 1 = -\sin x$.

$$\sin x + 1 = -\sin x$$
$$2\sin x = -1$$
$$\sin x = \frac{-1}{2}$$

Since sine has a period of 2π, find the solutions on the interval from 0 to 2π first (i.e. on the interval $[0, 2\pi)$).

$$x = \frac{7\pi}{6} \text{ or } x = \frac{11\pi}{6}$$

Every time we add a multiple of 2π to these solutions, we get another solution. Thus, we write the solution as

$$x = \frac{7\pi}{6} + 2n\pi \text{ or } x = \frac{11\pi}{6} + 2n\pi$$

Example: Solve $\tan^2 x - 3 = 0$.

$$\tan^2 x = 3$$
$$\sqrt{\tan^2 x} = \pm\sqrt{3}$$
$$\tan x = \pm\sqrt{3}$$

We had to isolate the trig function and then take the square root of both sides.

Because the period of tangent is π, we will first find all the solutions on the interval from 0 to π. (Use unit circle.)

$$x = \frac{\pi}{3} \text{ or } x = \frac{2\pi}{3}$$

Every time we add a multiple of π to either of these solutions we get an angle that has the same tangent. Thus, our solution is

$$x = \frac{\pi}{3} + n\pi \text{ or } x = \frac{2\pi}{3} + n\pi$$

Example: Solve $2\cos^2 x + \cos x - 1 = 0$ on the interval [0, 2π).

(Think of this as $2u^2 + u - 1 = 0$.)

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$2\cos x - 1 = 0 \text{ or } \cos x + 1 = 0$$

$$\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \text{or } \pi$$

Example: Solve $2\cos^2 x$ - $5\cos x$ - 3=0.

$$(2\cos x + 1)(\cos x - 3) = 0$$

$$2\cos x + 1 = 0, \cos x - 3 = 0$$

$$\cos x = \frac{-1}{2}, \cos x = -3$$

Since cos x cannot = -3, our only solution is $\frac{2\pi}{3}, \frac{4\pi}{3}$.

Example: Solve $2\sin 2t + 1 = 0$ on the interval $[0, 2\pi)$.

$$2\sin 2t + 1 = 0$$

$$2\sin 2t = -1$$

$$\sin 2t = \frac{-1}{2}$$

Ask the question, "The sine of what angle is $\frac{-1}{2}$?"

The answer is $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{31\pi}{6}, \ldots.$

So, $2t = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{31\pi}{6}, \ldots$

Since we are solving for t, divide through by 2.

$$t = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}, \frac{31\pi}{12}, \ldots$$

Since $0 \leq t < 2\pi$, we will limit our answer to

$$t = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

Example: Solve $\cos\frac{x}{3}=\frac{\sqrt{2}}{2}$ for $0 \le x < 2\pi$.

Ask "Where is the cosine equal to $\frac{\sqrt{2}}{2}$?"

The answer is at $\frac{\pi}{4},\frac{7\pi}{4},\frac{9\pi}{4},\frac{15\pi}{4},\frac{17\pi}{4},\frac{23\pi}{4},...$

$$\text{So,}\quad \frac{x}{3}=\frac{\pi}{4},\frac{7\pi}{4},\frac{9\pi}{4},\frac{15\pi}{4},\frac{17\pi}{4},\frac{23\pi}{4},...$$

Multiply through by 3 to solve for x.

$$x=\frac{3\pi}{4},\frac{21\pi}{4},\frac{27\pi}{4},\frac{45\pi}{4},\frac{51\pi}{4},\frac{69\pi}{4},...$$

Of these, the only one on the interval $0 \le x < 2\pi$ is $\frac{3\pi}{4}$.

So, the solution is $x=\frac{3\pi}{4}$