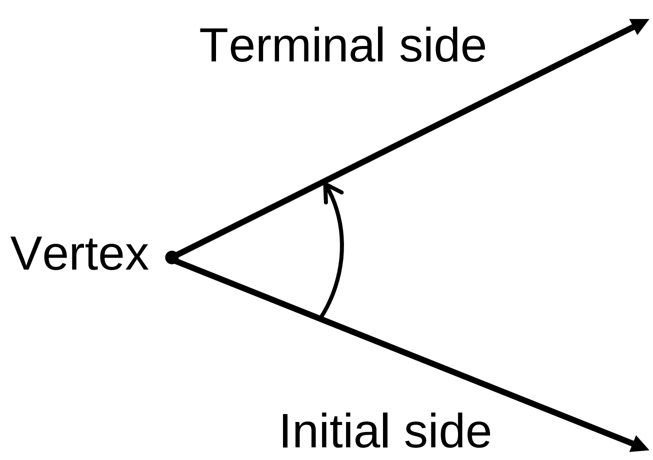
Radian and Degree Measure

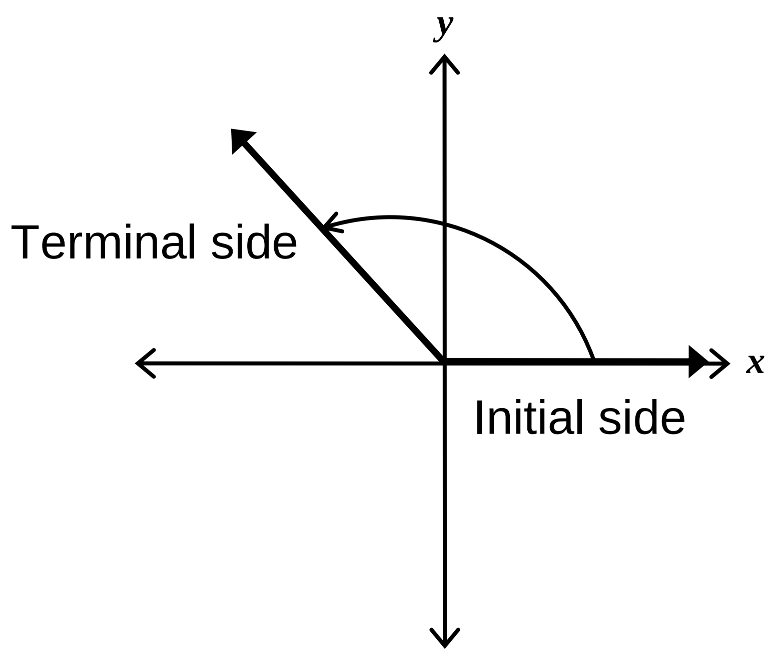
***Trigonometry** comes from the Greek word meaning "*measurement of triangles*." It primarily dealt with angles and triangles as it pertained to navigation, astronomy, and surveying. Today, the use has expanded to involve rotations, orbits, waves, vibrations, etc.

Definitions:

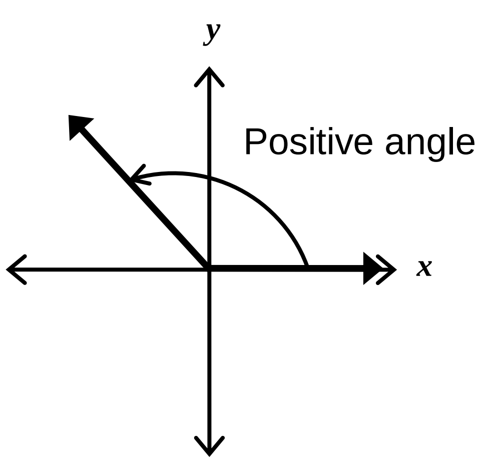
- An angle is determined by rotating a ray (half-line) about its endpoint.
- The initial side of an angle is the starting position of the rotated ray in the formation of an angle.
- The terminal side of an angle is the position of the ray after the rotation when an angle is formed.
- The vertex of an angle is the endpoint of the ray used in the formation of an angle.

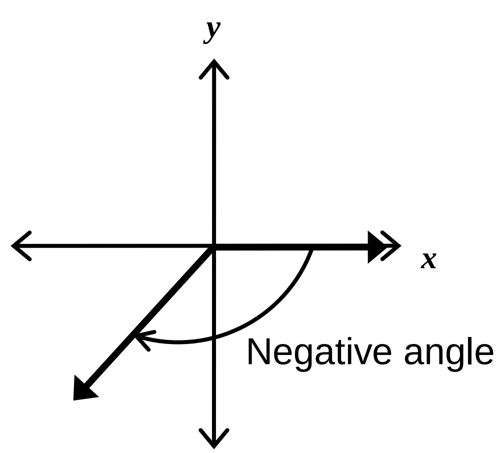


- An angle is in <u>standard position</u> when the angle's vertex is at the origin of a coordinate system and its initial side coincides with the positive *x*-axis.

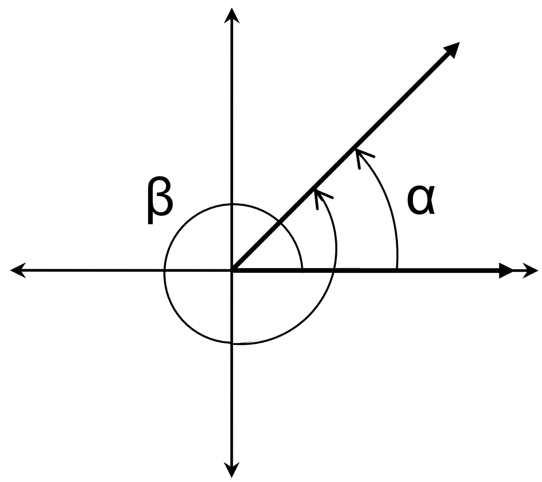


- A <u>positive angle</u> is generated by a counterclockwise rotation; whereas a <u>negative angle</u> is generated by a clockwise rotation.





- If two angles are coterminal, then they have the same initial side and the same terminal side.

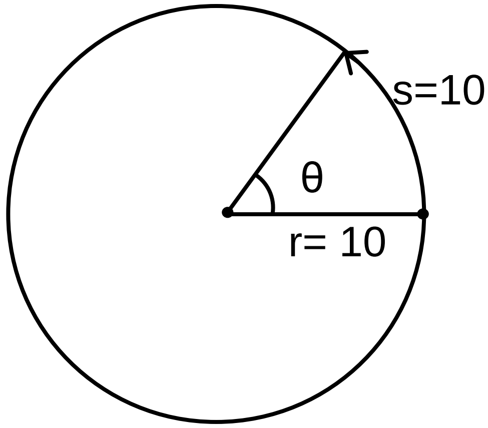


Radian Measure

Definitions:

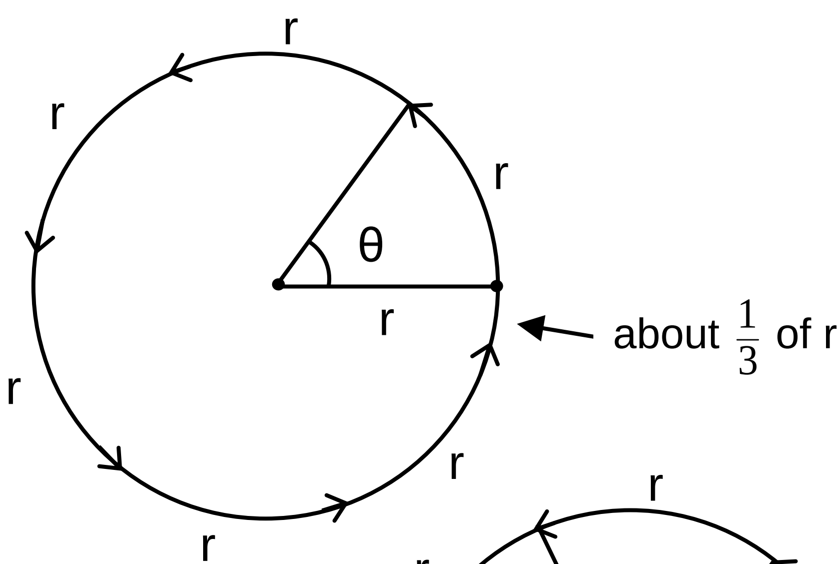
- The measure of an angle is determined by the amount of rotation from the initial side to the terminal side.
- A central angle is one whose vertex is the center of a circle.

- One radian is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle.

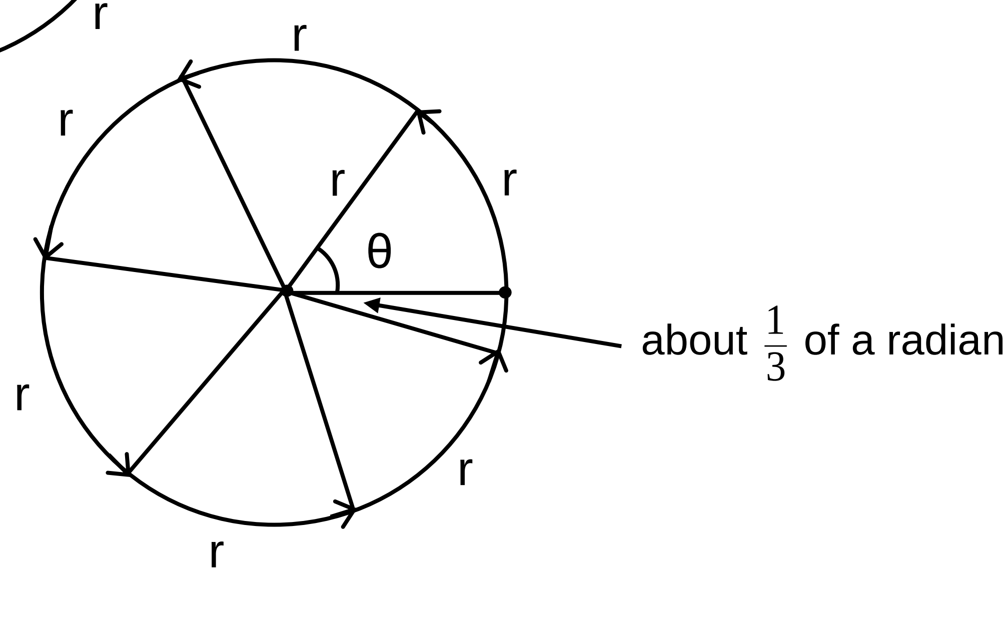


θ is <u>1 radian</u> in size.

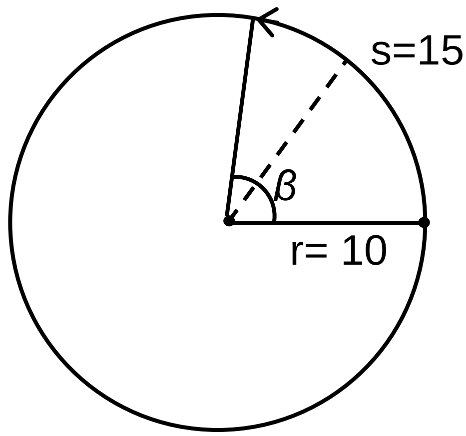
How many radians are in a circle?



θ is 1 radian



There are about 6 $\frac{1}{3}$ radians in a circle.



If s = 15, then we need to find out how many r's are in s to know the number of radians in β.

$\frac{15}{10}$ = 1.5, so β is 1.5 radians.

***In general**, the radian measure of a central angle θ with radius *r* and arc length *s* is

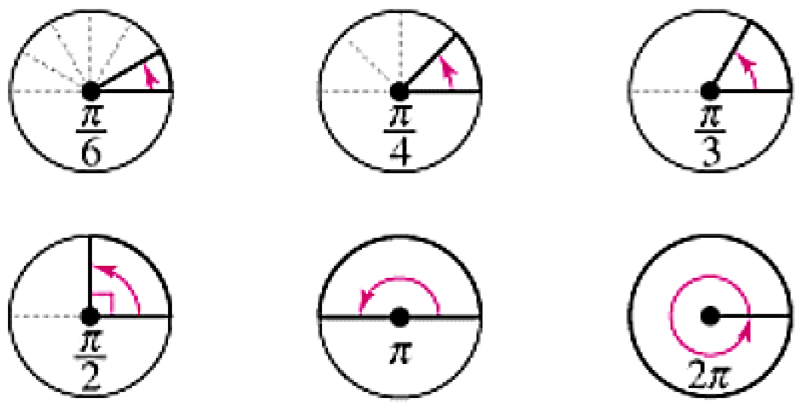
$$\theta = \frac{s}{r}$$

We know that the circumference of a circle is $2\pi r$. If we consider the arc *s* as being the circumference, we get

$$\theta = \frac{2\pi r}{r} = 2\pi$$

This means that the circle itself contains an angle of rotation of 2π radians. Since 2π is approximately 6.28, this matches what we found above. There are a little more than 6 radians in a circle. (2π to be exact.)

Therefore: A <u>circle</u> contains 2π radians.
A <u>semi-circle</u> contains π radians of rotation.
A <u>quarter of a circle</u> (which is a right angle) contains $\frac{\pi}{2}$ radians of rotation.



<u>Definition</u>: A <u>degree</u> is a unit of angle measure that is equivalent to the rotation in 1/360th of a circle.

Because there are 360° in a circle, and we now know that there are also 2π radians in a circle, then $2\pi = 360°$.

$360° = 2\pi$ radians	2π radians $= 360°$
$180° = \pi$ radians	1π radians $= 180°$
$1° = \frac{\pi}{180}$ radians	1 radian $= \frac{180°}{\pi}$

To convert <u>radians to degrees</u>, multiply by $\frac{180°}{\pi}$.

To convert <u>degrees to radians</u>, multiply by $\frac{\pi}{180}$.

Example: Convert 120° to radians.

$$120° = 120\left(\frac{\pi}{180}\right) = \frac{120\pi}{180} = \frac{2\pi}{3}$$

Example: Convert -315° to radians.

$$-315° = -315\left(\frac{\pi}{180}\right) = \frac{-315\pi}{180} = \frac{-7\pi}{4}$$

Example: Convert $\frac{5\pi}{6}$ to degrees.

$$\frac{5\pi}{6} = \frac{5\pi}{6}\left(\frac{180}{\pi}\right) = \frac{900}{6} = 150°$$

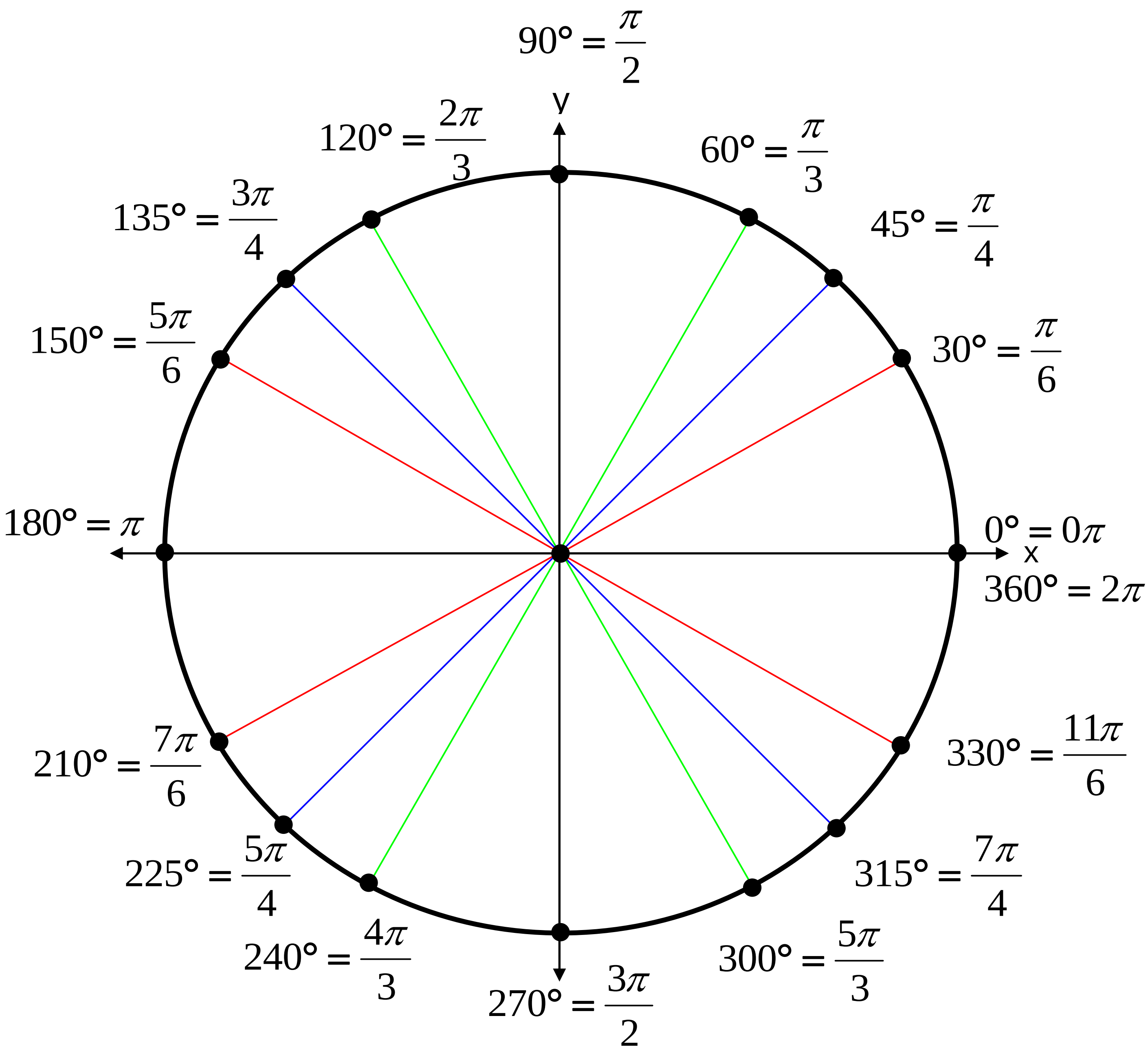
Example: Convert 7 to degrees.

$$7 = 7\left(\frac{180}{\pi}\right) = \frac{1260}{\pi} = 401.07°$$

This makes sense, because 7 radians would be a little more than a complete circle, and 401.07° is a little more that 360°

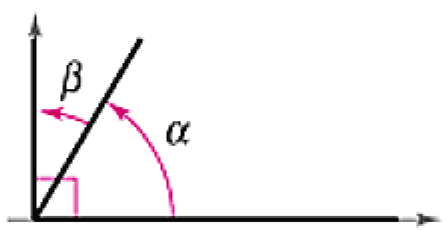
*Notice: If there is no unit specified, it is assumed to be radians.

Degree and Radian Equivalent measures

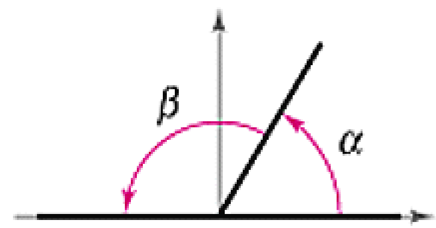


<u>Definition</u>: Two positive angles α and β are <u>complementary</u> if their sum is $\frac{\pi}{2}$ or 90°.

Two positive angles α and β are <u>supplementary</u> if their sum is π or 180°.



Complementary Angles



Supplementary Angles

<u>Definition</u>: An <u>acute angle</u> has a measure between 0 and $\frac{\pi}{2}$ (or between 0° and 90°.)

An <u>obtuse angle</u> has a measure between $\frac{\pi}{2}$ and π (or between 90° and 180°.)

Example: Find the supplement and complement of $\frac{\pi}{5}$.

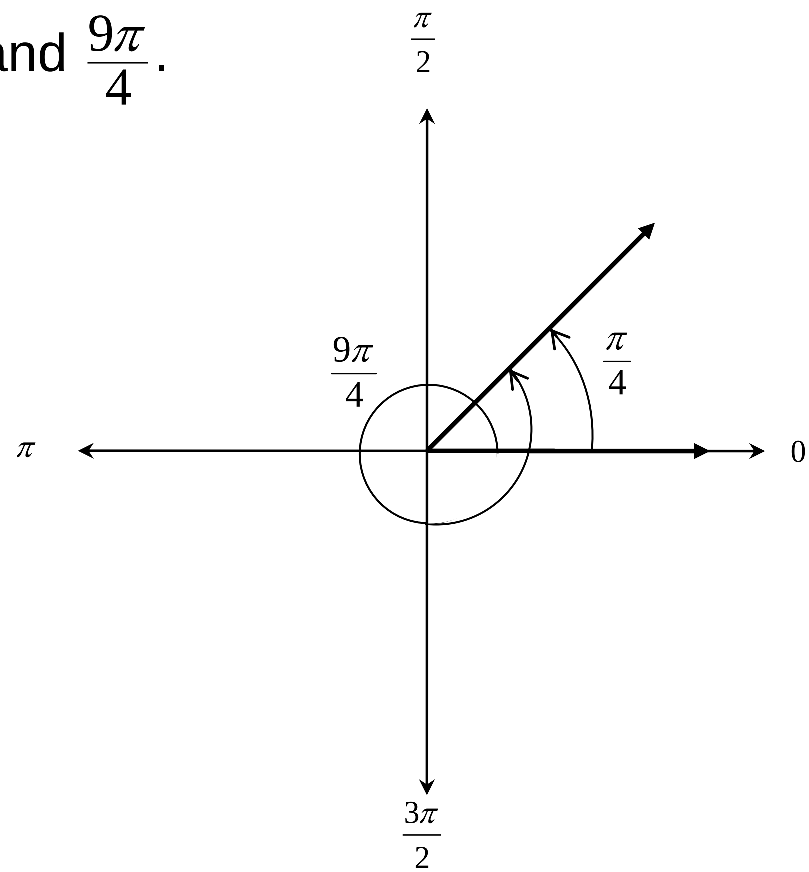
complement: $\frac{\pi}{2}-\frac{\pi}{5}=\frac{3\pi}{10}$

supplement: $\pi-\frac{\pi}{5}=\frac{4\pi}{5}$

Coterminal Angles

Two angles are coterminal if they have the same initial side and the same terminal side.

Look at $\frac{\pi}{4}$ and $\frac{9\pi}{4}$.

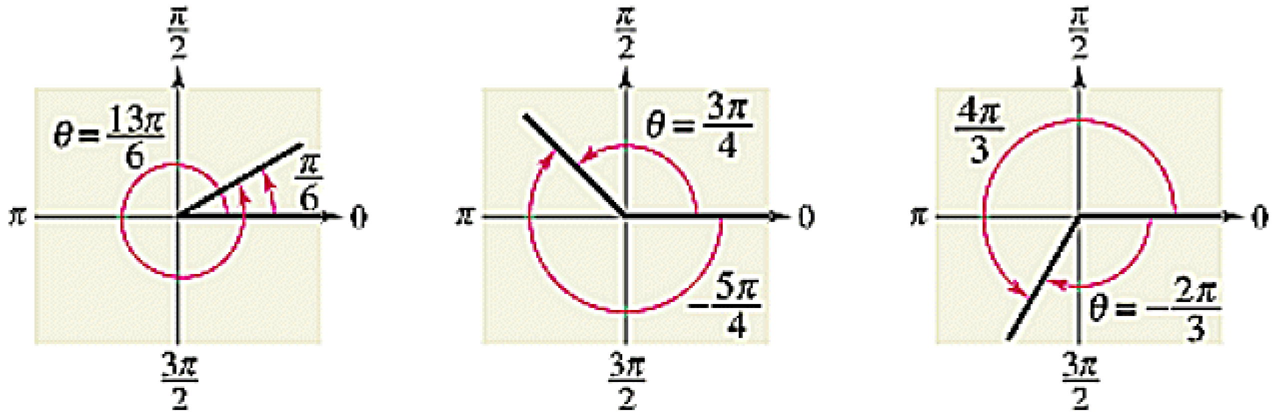


$\frac{9\pi}{4}$ is the same as $\frac{\pi}{4}$ with 2π added to it.

***In general**, you can find an angle that is coterminal with an angle θ by adding or subtracting multiples of 2π. (Each multiple of 2π is a full revolution around the circle.)

We write this as $\theta + 2k\pi$, where k is an integer.

Examples of coterminal anlges:



Example: Find an angle that is coterminal with $\theta = \frac{-\pi}{8}$.

One possible solution: $\frac{-\pi}{8} + 2\pi = \frac{-\pi}{8} + \frac{16\pi}{8} = \frac{15\pi}{8}$.

Example: Find an angle that is coterminal with $\theta = \frac{3\pi}{4}$.

One possible solution: $\frac{3\pi}{4} - 2\pi = \frac{3\pi}{4} - \frac{8\pi}{4} = \frac{-5\pi}{4}$.

Example: List all of the angles that are coterminal with $\frac{\pi}{6}$

answer: $\frac{\pi}{6} \pm 2k\pi$

Calculator Conversion

Fractional parts of degrees can be denoted as decimal degrees or as degrees, minutes and seconds.

$$1° = 60' \text{ (minutes)}$$
$$1' = 60'' \text{ (seconds)}$$

This also means that, $1' = \frac{1}{60}°$

$1'' = \frac{1}{60}'$ or $\frac{1}{3600}°$

example: 34° 15′ 30″

*To convert to decimal degrees:

$$34 + \frac{15}{60} + \frac{30}{3600} = 34.2583°$$

On your graphing calculator:

Enter 34° 15′ 30″. Use the [ANGLE] menu for ° and ′ and [ALPHA] [+] for the ″. Press [MATH] [►Dec] [ENTER] to convert to decimal degrees.

Example: Convert 68° 22′ 46″ to decimal degrees.

answer: 68.3794°

<u>To convert decimal degrees to degrees, minutes, and seconds (DMS):</u>

Enter the decimal degree. Press [ANLGE] [►DMS] [ENTER] to convert. Round the seconds to the nearest second.

Example: Convert 8.875° to degrees, minutes and seconds.

answer: 8° 52′ 30″

<u>Applications</u>

Because we already know that with radian measure $\theta = \frac{s}{r}$, where *s* is the arc length, then $s = r\,\theta$.

Example: Find the length of the arc that subtends a central angle with measure 120° in a circle with radius 5 inches.

Change to radians.

$$s = 5\left(120\left(\frac{\pi}{180}\right)\right) = \frac{10\pi}{3} \approx 10.47 \text{ inches}$$

Linear and Angular Speed

Consider a particle moving at a constant speed along a circular arc of radius *r*. If *s* is the length of the arc traveled in time *t*, then the **linear speed** of the particle is

$$\text{Linear speed} = \frac{\text{arc length}}{\text{time}} = \frac{s}{t} \text{ or } \frac{r\theta}{t}$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** of the particle is

$$\text{Angular speed} = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}$$

***Note:** Linear speed can also be represented as

$$\frac{s}{t} \text{ or } \frac{s}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) = (\text{radius})(\text{angular speed})$$

Example: The circular blade on a saw rotates at 2400 revolutions per minute.

(a) Find the angular speed in radians per second.

Because each revolution generates 2π radians, it follows that the saw turns (2400)(2π) = 4800 π radians per minute. In other words, the angular speed is

$$\text{Angular speed } \frac{\theta}{t} = \frac{4800\pi \text{ radians}}{60 \text{ seconds}} = 80\pi \text{ radians per second}$$

(b) The blade has a diameter of 16 inches. Find the linear speed of a blade tip.

$$\text{Linear speed} = \frac{s}{t}$$

If we use *t*=1, then from part (a) we know that θ is 80π because that is the radians in one second's rotation.

$$= \frac{r\theta}{t}$$

$$= \frac{(8 \text{ inches})(80\pi)}{(1 \text{ second})}$$

$$\approx 2010.6 \text{ inches/second}$$

Alternately,

$$\text{Linear speed} = r\left(\frac{\theta}{t}\right)$$

$$= (\text{radius})(\text{angular speed})$$

$$= (8 \text{ inches})(80\pi \text{ radians/second})$$

$$\approx 2010.6 \text{ inches/second}$$