section 4.2

**The Ruler Postulate guarantees that you can measure any segment.

**The Protractor Postulate guarantees that you can measure any angle.

Protractor Postulate: For every angle A there corresponds a positive real number less than or equal to 180.

$$0 < m\angle A \leq 180^\circ$$

Note: $m\angle A$ stands for the measure of $\angle A$

Measuring angles with Protractors:

protractor – instrument for measuring angles

degree measure – real number that represents the measure of an angle

degree – unit of measure (equals 1/180 of a semicircle)

**The <u>Completeness Postulate</u> guarantees that a segment exists for every measure

** The <u>Continuity Postulate</u> guarantees that an angle exists for any measure.

<u>Continuity Postulate</u>: If k is a half-plane determined by $\overleftrightarrow{AC}$, then for every real number, $0 < x \leq 180$, there is exactly one ray, $\overrightarrow{AB}$, that lies in k such that $m\angle BAC = x$

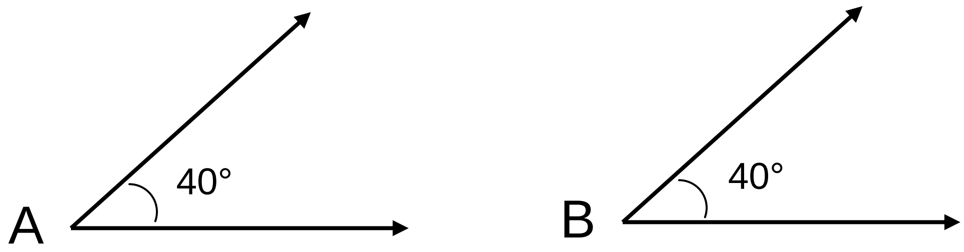
The Protractor Postulate and Continuity Postulate say the same thing but in reverse order. We call these statements <u>converses</u>.

Classifying Angles According to their Measure:

Angle name	**Angle measure**
acute angle	$0 < x < 90^{o}$
right angle	$x = 90^{o}$
obtuse angle	$90^{o} < x < 180^{o}$
straight angle	$x = 180^{o}$

Definition:

<u>congruent angles</u> – angles that have the same measure. **(**symbol: ≅)



$m\angle A = m\angle B$ so we say $\angle A \cong \angle B$

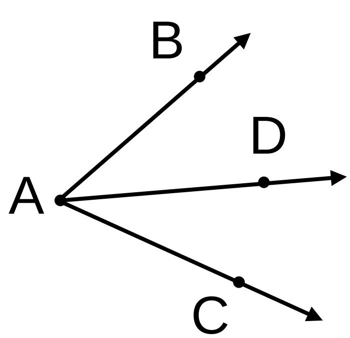
We use = because we are comparing numbers.

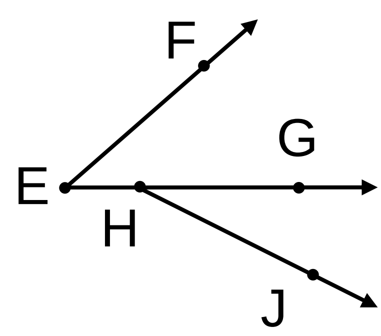
We use ≅ because we are comparing figures.

section 4.3

Definition:

<u>adjacent angles</u> - two coplanar angles that have a common side and a common vertex but no common interior points.

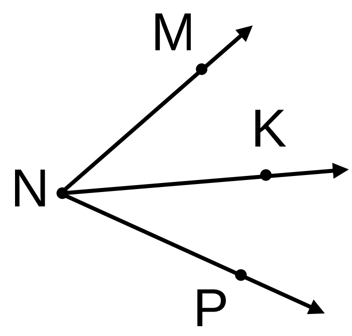




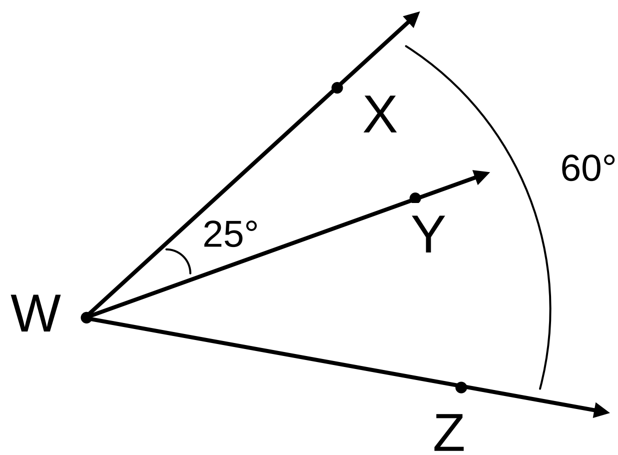
$\angle BAD$ and $\angle DAC$ are adjacent

$\angle EFG$ and $\angle GHJ$ are not adjacent

<u>Angle Addition Postulate:</u> If k lies in the interior of ∠MNP, then m∠MNP = m∠MNK + m∠KNP.



Example: Find m∠YWZ.

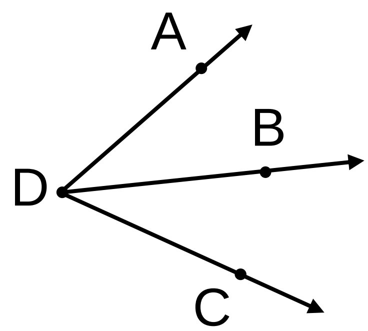


<u>Solution</u>: m∠YWZ = 60° - 25° = 35°

With segments we have midpoints. With angles we have bisectors.

Definition:

An <u>angle bisector</u> is a ray that (except for its origin) is in the interior of an angle and forms congruent adjacent angles.



$\overrightarrow{DB}$ bisects $\angle ADC$ so we have $\angle ADB \cong \angle BDC$

(i.e. $m\angle ADB = m\angle BDC$)

We can also say that: $m\angle ADB = \frac{1}{2}m\angle ADC$

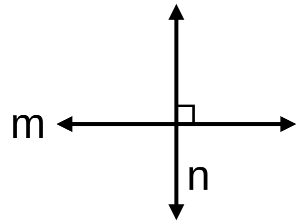
$m\angle BDC = \frac{1}{2}m\angle ADC$

<u>Question</u>: What happens if we bisect a straight angle?

We get 2 right angles

Definition:

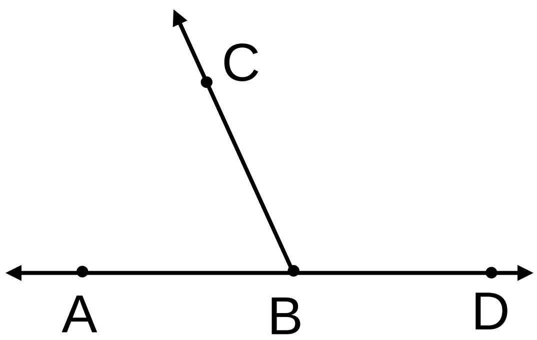
<u>perpendicular lines</u> – lines that intersect to form right angles (symbol: $\perp$)



$m \perp n$

Definition:

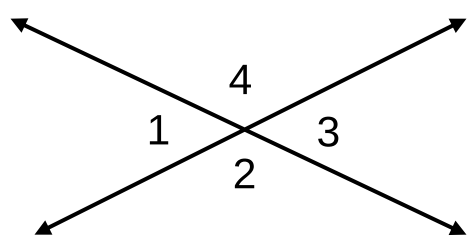
A <u>linear pair</u> is a pair of adjacent angles whose noncommon sides form a straight angle (are opposite rays)



$\angle$ABC and $\angle$CBD are a linear pair

Definition:

<u>Vertical angles</u> – angles adjacent to the same angle and forming linear pairs with it



$\angle$1 and $\angle$3 are vertical angles
$\angle$2 and $\angle$4 are vertical angles

Definitions:

Two angles are <u>complementary</u> if the sum of their angle measures is 90º.

Two angles are <u>supplementary</u> if the sum of their angle measures is 180º.

Sample Problems: Let m∠A = 58º. Find the following measures:

1. The supplement of ∠A.

   answer: 122º

2. The complement of ∠A.

   answer: 32º

3. The angle that makes a vertical angle with ∠A.

   answer: 58º

4. The angle that makes a linear pair with ∠A.

   answer: 122º

5. The angles formed when ∠A is bisected.

   answer: 29º

<u>Theorems</u>:

Theorem 4.1: All right angles are congruent.

Theorem 4.2: If two angles are adjacent and supplementary, then they form a linear pair.

Theorem 4.3: Angles that form a linear pair are supplementary.

Theorem 4.4: If one angle if a linear pair is a right angle, then the other angle is also a right angle.

Theorem 4.5: *<u>Vertical Angle Theorem</u>*. Vertical angles are congruent.

Theorem 4.6: Congruent supplementary angles are right angles.

Theorem 4.7: *<u>Angle Bisector Theorem</u>*. If $\overrightarrow{AB}$ bisects $\angle CAD$, then $m\angle CAB = \frac{1}{2}m\angle CAD$

<u>Note</u>: Many diagrams are not intended to be accurate, so be careful!!! For example, just because two lines "look" parallel, don't assume they are.

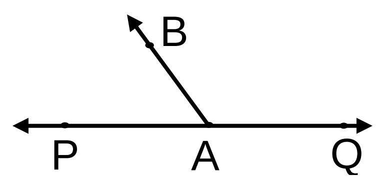
Proof of Theorem 4.1: All right angles are congruent.

Given: ∠A and ∠B are right angles
Prove: ∠A ≅ ∠B

Statements	Reasons
1. ∠A & ∠B are right angles	1. Given
2. m∠A = 90º, m∠B = 90º	2. defn. of right angle
3. m∠A = m∠B	3. substitution
4. ∠A ≅ ∠B	4. defn. of congruent ∠s

Proof of Theorem 4.3: Angles that form a linear pair are supplementary.

Given: ∠PAB and ∠BAQ form a linear pair
Prove: ∠PAB and ∠BAQ are supplementary



Statements	Reasons
1. ∠PAB and ∠BAQ form a linear pair	1. Given
2. ∠PAB and ∠BAQ are adjacent and ∠PAQ is straight	2. defn. of linear pair
3. m∠PAB + m∠BAQ = m∠PAQ	3. Angle Add. Post.
4. m∠PAQ = 180º	4. defn. of straight angle
5. m∠PAB + m∠BAQ =180º	5. substitution
6. ∠PAB and ∠BAQ are supplementary	6. defn. of supp. angles

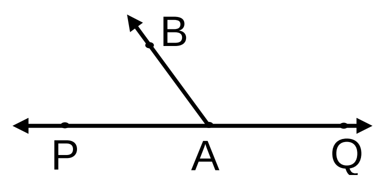
Proof of Theorem 4.1: All right angles are congruent.

Given: ∠A and ∠B are right angles
Prove: ∠A ≅ ∠B

Statements	Reasons
1. ∠A & ∠B are right angles	1.
2. m∠A = 90º, m∠B = 90º	2.
3. m∠A = m∠B	3.
4. ∠A ≅ ∠B	4.

Proof of Theorem 4.3: Angles that form a linear pair are supplementary.

Given: ∠PAB and ∠BAQ form a linear pair
Prove: ∠PAB and ∠BAQ are supplementary



Statements	Reasons
1. ∠PAB and ∠BAQ form a linear pair	1.
2. ∠PAB and ∠BAQ are adjacent and ∠PAQ is straight	2.
3. m∠PAB + m∠BAQ = m∠PAQ	3.
4. m∠PAQ = 180º	4.
5. m∠PAB + m∠BAQ =180º	5.
6. ∠PAB and ∠BAQ are supplementary	6.

Triangles

Classifying triangles by their angles:

An acute triangle has 3 acute angles.

A right triangle has a right angle.

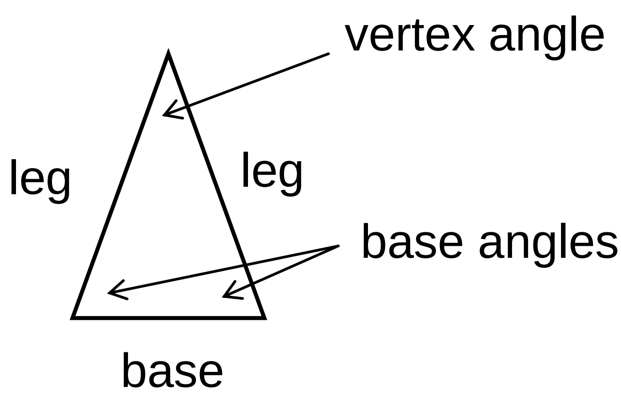
An obtuse triangle has an obtuse angle.

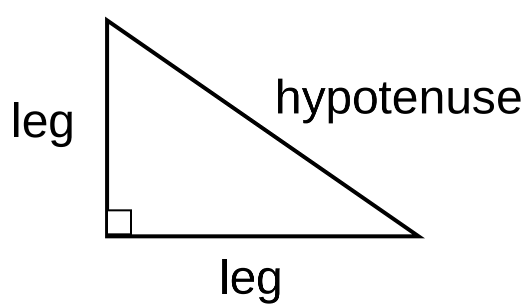
Classifying triangles by the length of their sides:

A scalene triangle has no congruent sides.

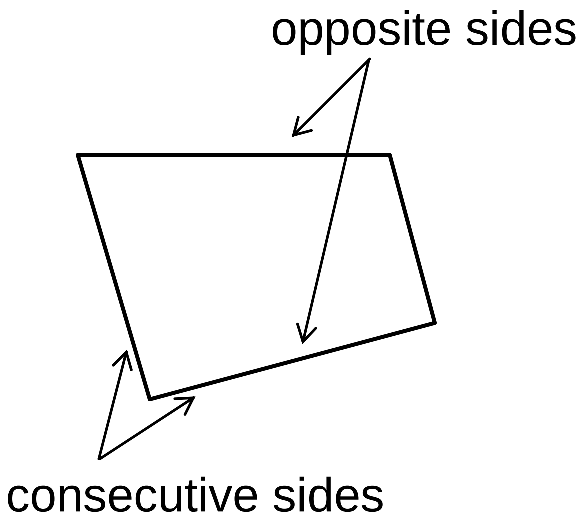
An isosceles triangle has at least 2 congruent sides.

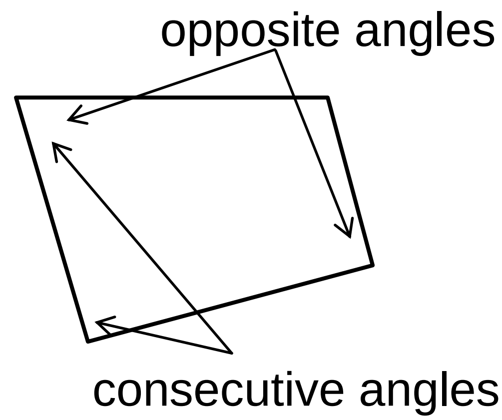
An equilateral triangle has an 3 congruent sides.





Quadrilaterals





Definitions:

opposite sides of a quadrilateral – segments that have no points in common

consecutive sides of a quadrilateral – 2 sides that intersect

opposite angles of a quadrilateral – angles whose vertices are not the endpoints of the same side

consecutive angles of a quadrilateral – angles whose vertices are endpoints of the same side

Classifying Quadrilaterals:

Definitions:

<u>trapezoid</u> – quadrilateral with a pair of parallel opposite sides (It could also have 2 pairs of parallel opposite sides.)

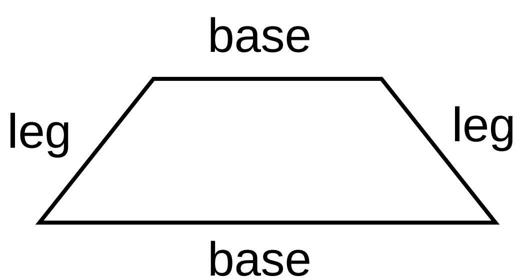
<u>parallelogram</u> – quadrilateral with 2 pairs of parallel opposite sides

<u>rectangle</u> – a parallelogram with 4 right angles

<u>rhombus</u> – a parallelogram with 4 congruent sides

<u>square</u> – a rectangle with 4 congruent sides (or a rhombus with 4 congruent angles)

Note: Some texts define trapezoids as quadrilaterals with only ***<u>one</u>*** pair of parallel opposite sides.



Definition:

<u>isosceles trapezoid</u> – one whose legs are congruent